Binary Search

The main concept for both the Iterative & Recursive approach is same.

Only Criteria : The array should be in ascending order.

target element

Array :	36	48	62	146	152	163	235	237	259	348
Index :	0	1	2	3	4	5	6	7	8	9

Step 1 : Find the value of the middle index.

middle = (left + right) // 2 , which means the quotient.

Step 2 : Check whether array[middle] == target element.

If YES, return middle as our index.
If NO, check further.

target_element present at the left of the array[middle]

$\Downarrow$

arr[middle] > target_element

$\Downarrow$

We will ignore what happens to the right hand side of the arr[middle] and change the array indices to
right = middle , left unchanged

target_element present at the right of the array[middle]

$\Downarrow$

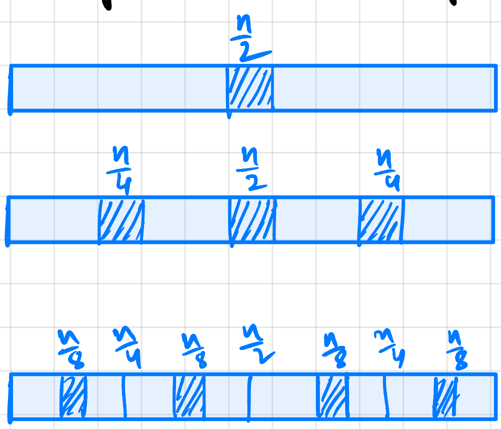
arr[middle] < target_element

$\Downarrow$

We will ignore what happens to the left hand side of the arr[middle] and change the array indices to
left = middle + 1, right unchanged.

Repeat Step 1 and Step 2 until found or not found.

Binary Search Analysis



No. of positions	No. of comparisons.
1	1
2	2
4	3

Average Comparisons :

$$\frac{1}{n}\left[\left(1.1+2.2+4.3+8.4+\cdots+2^{\lg n-1}.\lg n\right)+\lg n\right]$$

Let $S = 1.1 + 2.2 + 4.3 + 8.4 + \cdots + 2^{\lg n-1}.\lg n$.

$\therefore$ Avg comparisons $= \dfrac{S+\lg n}{n}$.

Now, S is an AGP series. Solving the AGP series to obtain S.

$$S = 1.1 + 2.2 + 4.3 + 8.4 + \cdots + 2^{\lg n-1}.\lg n$$

$$2S = 2.1 + 4.2 + 8.3 + \cdots + 2^{\lg n-1}.(\lg n-1) + 2^{\lg n}.\lg n.$$

$$\therefore S-2S = 1 + \left[2(2-1)+4(3-2)+8(4-3)+\cdots+2^{\lg n-1}\right] - 2^{\lg n}.\lg n.$$

$$\Rightarrow -S = 1+\left[2+4+8+\cdots+2^{\lg n-1}\right] - n\lg n.$$

$$\Rightarrow -S = 1+\left[2\left(1+2+4+\cdots+2^{\lg n-2}\right)\right] - n\lg n.$$

$$\Rightarrow -S = 1 + 2.\frac{2^{\lg n-2+1}-1}{(2-1)} - n\lg n$$

$$\Rightarrow -S = 1+2\left(2^{\lg n-1}-1\right) - n\lg n$$

$$\Rightarrow -S = 1+2^{\lg n}-2-n\lg n = n-n\lg n-1$$

$$\Rightarrow S = n\lg n - n + 1.$$

$$\therefore \text{Avg Comp} = \frac{S+\lg n}{n} = \frac{n\lg n - n + 1 + \lg n}{n} = O(\lg n).$$

$$\Rightarrow \boxed{\text{Avg Comp} = O(\lg n)}$$